CH 9 – RADICALS

We've learned that there is a positive real number whose square is 3, and that this number is called the positive square root of 3. We also have seen that this number is written with a 3 inside a radical sign. This chapter discusses the various operations involving these "radicals."

$\sqrt{3}$

❑ All Kinds of Roots

The number 9 has two ***square roots***, 3 and –3. This is because 3 squared is 9 and –3 squared is 9. The positive square root of 9 (the 3) is denoted $\sqrt{9}$, and the negative square root of 9 (the –3) is written $-\sqrt{9}$. In other words, $\sqrt{9} = 3$, and only 3, while $-\sqrt{9} = -3$.

In analyzing $\sqrt{-25}$, the positive square root of –25, we discover that we cannot find an answer for this problem, since the square of a real number can never be negative. If there is an answer to $\sqrt{-25}$, it lies outside $\mathbb{R}$.

Consider the number 8. Since $2^3 = 8$, we can say that 2 is a ***cube root*** of 8. In fact, it's the only cube root of 8, simply because there's no other real number whose cube is 8. Perhaps a little surprising is that we can calculate the cube root of a negative number without leaving $\mathbb{R}$. For example, $\sqrt[3]{-27} = -3$, since $(-3)^3 = -27$.

The number 16 has two ***fourth roots***. The positive fourth root is $\sqrt[4]{16} = 2$, and the negative fourth root is $-\sqrt[4]{16} = -2$. After all, both 2 and –2 raised to the fourth power result in 16. However, just like square roots, $\sqrt[4]{-1}$ is not a real number.

The ***fifth root*** of 32 is 2; that is, $\sqrt[5]{32} = 2$ -- this is because $2^5 = 32$. Like cube roots, we can calculate the fifth root of a negative number. For example, $\sqrt[5]{-243}$ equals –3, since $(-3)^5 = -243$.

Consider the expression $\sqrt[3]{ab}$. The entire expression can be called a **radical**. The symbol $\sqrt{}$ is called the **radical sign**, and the quantity ab is called the **radicand**.

A Final Comment: The number 23 has two square roots, $\sqrt{23}$ and $-\sqrt{23}$. Not only do these square roots not result in an integer, they actually result in irrational numbers. Similarly, each of the following is an irrational number: $\sqrt[3]{17}$, $\sqrt[4]{123}$, and $\sqrt[5]{-1001}$ -- each of them is an infinite, non-repeating decimal.

Homework

1. Find the square root(s) of

   a. 100 b. 15 c. 0 d. –36 e. 1

2. Find the cube root(s) of

   a. 64 b. –125 c. 0 d. 20 e. 1

3. Find the fourth root(s) of

   a. 81 b. 0 c. –625 d. 25 e. 1

4. Find the fifth root(s) of

   a. 1 b. 0 c. –243 d. 29 e. 32

❑ Simplifying Radicals

During our discussion of radicals, we will assume that all variables are non-negative (positive or zero). Therefore, even though $\sqrt{x^2} = |x|$ when x is an arbitrary real number, our restriction to non-negative values implies that we can dispense with the absolute value signs: If $x \geq 0$, then $\sqrt{x^2} = x$.

The basic rule regarding radicals is contained in the following rule.

THEOREM: Assume that x and y represent non-negative numbers. Then

$$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$$

The root of a product
is the product of the roots.

Proof: Our technique will be to convert the left side of the formula to exponent form, use a law of exponents, and then convert back to radical form to arrive at the right side of the formula.

	$\sqrt[n]{xy}$	
=	$(xy)^{1/n}$	(definition of fractional exponent)
=	$x^{1/n}\,y^{1/n}$	(law of exponents)
=	$\sqrt[n]{x}\,\sqrt[n]{y}$	(definition of fractional exponent)

Q.E.D.

THEOREM: If x represents a non-negative number, then

$$\sqrt[n]{x^n} = x$$

The *n*th root cancels the *n*th power.

Proof: Our method of proof is similar to the previous theorem.

$$\sqrt[n]{x^n} = (x^n)^{1/n} = x^{n\cdot\frac{1}{n}} = x^1 = x$$ Q.E.D.

EXAMPLE 1: **Simplify each radical expression:**

A. $\sqrt{200} = \sqrt{100\times 2} = \sqrt{100}\times\sqrt{2} = 10\sqrt{2}$

B. $\sqrt[3]{81} = \sqrt[3]{27\times 3} = \sqrt[3]{27}\times\sqrt[3]{3} = 3\sqrt[3]{3}$

C. $\sqrt[4]{1250} = \sqrt[4]{625\cdot 2} = \sqrt[4]{625}\sqrt[4]{2} = 5\sqrt[4]{2}$

D. $\sqrt[3]{2250}$ Sometimes the radicand (the 2250) is too big to easily see if there's a perfect cube in it. So let's try a slightly different approach. We factor the 2250 into primes to get

$$2250 = 2\cdot 3^2\cdot 5^3.$$

Clearly we can take the cube root of 5^3 (it's 5), but there are not enough of the other factors to take their cube roots. So we can write

$$\sqrt[3]{2250} = \sqrt[3]{2\cdot 3^2\cdot 5^3} = 5\sqrt[3]{2\cdot 3^2} = 5\sqrt[3]{18}$$

EXAMPLE 2: **Simplify each radical expression:**

A. $\sqrt{50xy^2} = \sqrt{5^2 \cdot 2 \cdot x \cdot y^2} = 5y\sqrt{2x}$

B. $\sqrt{144a^{46}b^{19}} = \sqrt{12^2 \cdot (a^{23})^2 \cdot (b^9)^2 \cdot b} = 12a^{23}b^9\sqrt{b}$

C. $\sqrt[3]{16x^{48}w^{14}} = \sqrt[3]{2^3 \cdot 2 \cdot (x^{16})^3 \cdot (w^4)^3 \cdot w^2} = 2x^{16}w^4\sqrt[3]{2w^2}$

D. $\sqrt[4]{c^{20}d^{39}} = \sqrt[4]{(c^5)^4(d^9)^4 d^3} = c^5d^9\sqrt[4]{d^3}$

E. $\sqrt[5]{x^4y^{53}} = \sqrt[5]{x^4(y^{10})^5 y^3} = y^{10}\sqrt[5]{x^4y^3}$

F. $\sqrt[3]{54a^2b^{27}c^{17}} = \sqrt[3]{3^3 \cdot 2 \cdot a^2 \cdot (b^9)^3 \cdot (c^5)^3 \cdot c^2} = 3b^9c^5\sqrt[3]{2a^2c^2}$

G. $\sqrt[5]{64x^3y^5z^{25}w^{47}} = \sqrt[5]{2^5 \cdot 2 \cdot x^3 \cdot y^5 \cdot (z^5)^5 \cdot (w^9)^5 \cdot w^2}$
$= 2yz^5w^9\sqrt[5]{2x^3w^2}$

As we'll see in the following example, fractional exponents are indispensable for some radical problems.

EXAMPLE 3: **Simplify each radical expression:**

A. $\sqrt{\sqrt[3]{x}} = \left(x^{1/3}\right)^{1/2} = x^{1/6} = \sqrt[6]{x}$

B. $\sqrt[4]{\sqrt[3]{\sqrt{2}}} = \left[\left(2^{1/2}\right)^{1/3}\right]^{1/4} = 2^{1/24} = \sqrt[24]{2}$

C. $\sqrt[12]{a^9} = \left(a^9\right)^{1/12} = a^{9/12} = a^{3/4} = \sqrt[4]{a^3}$

Homework

5. Simplify: a. $\sqrt{200xy^2z^3w^{16}v^{37}}$ b. $\sqrt{72a^{25}b^{12}}$

6. Simplify: a. $\sqrt[3]{16ab^2c^3d^{27}e^{50}}$ b. $\sqrt[3]{54x^{22}y^{32}}$

7. Simplify: a. $\sqrt[4]{32x^4y^{16}z^{39}}$ b. $\sqrt[4]{81a^4b^{196}}$

8. Simplify: a. $\sqrt[5]{64u^{32}v^{54}}$ b. $\sqrt[5]{486w^4z^{32}}$

9. Use your calculator to verify Example 3B.

10. DISprove the conjecture: $\sqrt{a+b} = \sqrt{a}+\sqrt{b}$ [Hint: use numbers]

11. Simplify each nested radical:

a. $\sqrt{\sqrt[4]{7}}$ b. $\sqrt[3]{\sqrt{10}}$ c. $\sqrt[5]{\sqrt[7]{x}}$

d. $\sqrt[4]{\sqrt[4]{n}}$ e. $\sqrt[5]{\sqrt[3]{\sqrt{a}}}$ f. $\sqrt[4]{\sqrt[3]{\sqrt{x}}}$

12. Simplify:

a. $\sqrt[6]{x^2}$ b. $\sqrt[8]{n^6}$ c. $\sqrt[10]{x^5}$ d. $\sqrt[9]{z^3}$

e. $\sqrt[20]{a^{15}}$ f. $\sqrt[21]{n^{14}}$ g. $\sqrt[30]{m^{26}}$ h. $\sqrt[88]{x^{77}}$

13. Simplify $\sqrt{\sqrt[3]{\sqrt[4]{3}}}$. Use your calculator to verify your answer.

❑ Adding and Subtracting Radicals

To add (or subtract) radicals, we can use either the concept of like terms or the idea of factoring.

EXAMPLE 4: **Simplify each radical expression:**

A. $3\sqrt{7}+5\sqrt{7} = 8\sqrt{7}$

They're like terms, so we can add them up, just like $3x + 5x = 8x$.
Or -- factor out the GCF: $\sqrt{7}(3+5) = 8\sqrt{7}$

B. $18\sqrt[3]{10}-20\sqrt[3]{10} = -2\sqrt[3]{10}$

C. $\sqrt{3}+\sqrt{5}$ cannot be worked out. They're unlike terms, and no amount of simplification will turn them into like terms.

D. $3\sqrt{20}+10\sqrt{45}$ It might appear that these terms cannot be combined, but don't jump to conclusions -- a little simplification first will yield an answer:

$$3\sqrt{20}+10\sqrt{45} = 3\sqrt{4\cdot 5}+10\sqrt{9\cdot 5} = 3\cdot 2\sqrt{5}+10\cdot 3\sqrt{5}$$
$$= 6\sqrt{5}+30\sqrt{5} = 36\sqrt{5}$$

E. $\sqrt{7}+\sqrt[3]{7}$ can't be simplified. Even though the radicands (the 7's) are the same, the roots are different, and therefore they are unlike terms and cannot be added together.

Homework

14. Simplify each radical expression:

a. $2\sqrt{x}+10\sqrt{x}$ b. $3\sqrt{7}-2\sqrt{2}$ c. $10\sqrt[3]{n}+3\sqrt[3]{n}$

d. $6\sqrt{x}-2\sqrt[3]{x}$ e. $9\sqrt{8}+10\sqrt{50}$ f. $\sqrt[3]{24}+\sqrt[3]{81}$

g. $3\sqrt{80}-5\sqrt{45}$ h. $10\sqrt[3]{189}+\sqrt[3]{56}$ i. $\sqrt[3]{z^2}+2\sqrt[3]{z^2}$

❑ Multiplying Radicals

Using the formula $\sqrt[n]{x}\sqrt[n]{y} = \sqrt[n]{xy}$, we can multiply "like roots." For example:

$$\sqrt[5]{3}\sqrt[5]{7} = \sqrt[5]{21}$$

Here's another example, with variables and simplifying at the end:

$$\sqrt[7]{x^5}\sqrt[7]{x^4} = \sqrt[7]{x^9} = \sqrt[7]{x^7x^2} = x\sqrt[7]{x^2}$$

Homework

15. Find the product:

a. $\sqrt{14}\sqrt{6}$ b. $\sqrt[3]{3}\sqrt[3]{9}$ c. $\sqrt{x^3}\sqrt{x^5}$

d. $\sqrt[3]{x^2y^5}\sqrt[3]{xy^8}$ e. $\sqrt{14}\sqrt{2}$ f. $\sqrt[3]{10}\sqrt[3]{12.5}$

g. $\sqrt[4]{2}\sqrt[4]{8}$ h. $\sqrt{x}\sqrt{yz}$ i. $\sqrt[3]{2}\sqrt[3]{4}$

16. Find the product:

a. $\sqrt{n}\left(1+\sqrt{n}\right)$ b. $\left(x+\sqrt{y}\right)\left(x-\sqrt{y}\right)$ c. $\left(2+\sqrt{3}\right)^2$

d. $\left(\sqrt{p}-q\right)^2$ e. $\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)$ f. $\left(x-\sqrt{n}\right)^2$

g. $\left(\sqrt{a}+\sqrt{c}\right)\left(\sqrt{a}-\sqrt{c}\right)$ h. $2\left(\sqrt{3}+\sqrt{4}\right)$ i. $\left(\sqrt{x}+\sqrt{x}\right)^2$

❑ Dividing Radicals

First we state and prove a theorem that we'll need later when we derive the Quadratic Formula.

THEOREM: Assume a and b are positive real numbers. Then

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

PROOF:

$$\sqrt{\frac{a}{b}}$$

$$= \left(\frac{a}{b}\right)^{1/2} \quad \text{(convert radical to fractional exponent)}$$

$$= \frac{a^{1/2}}{b^{1/2}} \quad \text{(a law of exponents)}$$

$$= \frac{\sqrt{a}}{\sqrt{b}} \quad \text{(convert back to radical form)} \quad \text{Q.E.D.}$$

Recall from Intermediate Algebra the following two procedures for removing a radical from the denominator of a fraction, called ***rationalizing the denominator***.

EXAMPLE 5:

A. $$\frac{3}{\sqrt{x}} = \frac{3}{\sqrt{x}} \underbrace{\left[\frac{\sqrt{x}}{\sqrt{x}}\right]}_{=1} = \frac{3\sqrt{x}}{\sqrt{x}\sqrt{x}} = \frac{3\sqrt{x}}{x}$$

B. $$\frac{\sqrt{2}}{5-\sqrt{10}} = \frac{\sqrt{2}}{5-\sqrt{10}} \underbrace{\left[\frac{5+\sqrt{10}}{5+\sqrt{10}}\right]}_{=1} = \frac{5\sqrt{2}+\sqrt{20}}{25-10} = \frac{5\sqrt{2}+2\sqrt{5}}{15}$$

[The quantity $5+\sqrt{10}$ is called the ***conjugate*** of $5-\sqrt{10}$.]

Homework

17. Simplify each radical expression:

a. $\frac{2}{\sqrt{8}}$ b. $\frac{x^2}{\sqrt{x}}$ c. $\frac{3}{5+\sqrt{2}}$ d. $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

e. $\frac{1}{\sqrt{3}}$ f. $\frac{6}{\sqrt{9}}$ g. $\frac{8}{\sqrt{2}}$ h. $\frac{\sqrt{x}}{\sqrt{y}}$

i. $\frac{5}{1+\sqrt{5}}$ j. $\frac{1+\sqrt{2}}{1-\sqrt{2}}$ k. $\frac{7}{3\sqrt{2}}$ l. $\frac{x}{n\sqrt{x}}$

m. $\frac{7}{1+\sqrt{7}}$ n. $\frac{a+\sqrt{b}}{a-\sqrt{b}}$ o. $\frac{a}{b\sqrt{c}}$ p. $\frac{7+\sqrt{103}}{7+\sqrt{103}}$

18. Two of the following are valid, while two of them are bogus. Which are which?

a. $\sqrt{a+b} = \sqrt{a}+\sqrt{b}$ b. $\sqrt{a-b} = \sqrt{a}-\sqrt{b}$

c. $\sqrt{ab} = \sqrt{a}\sqrt{b}$ d. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Practice Problems

19. a. Find the two fourth roots of 256.
b. Find all the cube roots of –1000.
c. Find all the square roots of –121.

20. Simplify each expression:

a. $\sqrt{7}\sqrt{5}$ b. $\sqrt{7}+\sqrt{5}$ c. $\dfrac{\sqrt{7}}{\sqrt{5}}$ d. $\sqrt{7}-\sqrt{5}$

e. $\sqrt{2}+\sqrt[3]{2}$ f. $\sqrt{28}+\sqrt{175}$ g. $\left(\sqrt{h}+7\right)^2$ h. $\left(3-\sqrt{t}\right)^2$

21. Simplify: $\sqrt[3]{108x^7y^{27}z^{50}}$

22. Write as a single radical: a. $\sqrt[7]{\sqrt{\sqrt[6]{x}}}$ b. $\sqrt{\sqrt[3]{\sqrt{x}}}$

23. Rationalize the denominator:

a. $\dfrac{4}{\sqrt{35}-5}$ b. $\dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$ c. $\dfrac{12}{\sqrt{9}+3}$

24. Simplify each expression:

a. $\sqrt{18}\sqrt{8}$ b. $\sqrt{7}-\sqrt{7}$ c. $\dfrac{1+\sqrt{7}}{\sqrt{7}}$ d. $\sqrt{50}-\sqrt{8}$

25. Simplify: $\sqrt[4]{64x^7y^{27}z^{50}}$

26. Write as a single radical: $\sqrt[4]{\sqrt{\sqrt[3]{x}}}$

27. Simplify: $\sqrt[10]{x^5}\ \sqrt[14]{x^7}$

28. Rationalize the denominator:

a. $\dfrac{3}{\sqrt{28}+5}$ b. $\dfrac{\sqrt{a}+b}{\sqrt{a}-b}$ c. $\dfrac{3-\sqrt{25}}{\sqrt{25}-3}$

29. Let $r_1 = \dfrac{3+\sqrt{11}}{4}$ and $r_2 = \dfrac{3-\sqrt{11}}{4}$.

Prove a. $r_1+r_2 = \dfrac{3}{2}$ b. $r_1r_2 = -\dfrac{1}{8}$

30. True/False:

a. 36 has two square roots.

b. $\sqrt{49} = \pm 7$

c. If $x \le 0$, then $\sqrt{x}$ is not a real number.

d. 64 has exactly one cube root.

e. $-\sqrt[4]{20}$ is a real number.

f. $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$

g. $\sqrt{ab} = \sqrt{a}\sqrt{b}$

h. For any real number x, $\sqrt{x^2} = x$.

i. $\sqrt[3]{18} = 3\sqrt{2}$

j. $\sqrt[5]{32n^{32}} = 2n^2$

k. $\sqrt{\sqrt[5]{\sqrt{a}}} = \sqrt[9]{a}$

l. $\sqrt[14]{x^{21}} = x\sqrt{x}$

m. $\sqrt{20} + \sqrt{5} = 5$

n. $\sqrt{20} + \sqrt{5} = 3\sqrt{5}$

o. $\left(\sqrt{a} + \sqrt{b}\right)^2 = a + b + 2\sqrt{ab}$

p. $\dfrac{7}{\sqrt{n}} = 7\sqrt{n}$

q. $\dfrac{1}{\sqrt{n}+1} = \dfrac{\sqrt{n}-1}{n-1}$

r. $\sqrt[3]{x^2} + \sqrt[3]{x} = x$

Solutions

1. a. ± 10 b. $\pm\sqrt{15}$ c. 0 d. $\varnothing$ e. ± 1

2. a. 4 b. -5 c. 0 d. $\sqrt[3]{20}$ e. 1

3. a. ± 3 b. 0 c. $\varnothing$ d. $\pm\sqrt[4]{25}$ e. ± 1

4. a. 1 b. 0 c. -3 d. $\sqrt[5]{29}$ e. 2

5. a. $10yzw^8v^{18}\sqrt{2xzv}$ b. $6a^{12}b^6\sqrt{2a}$

6. a. $2cd^9e^{16}\sqrt[3]{2ab^2e^2}$ b. $3x^7y^{10}\sqrt[3]{2xy^2}$

7. a. $2xy^4z^9\sqrt[4]{2z^3}$ b. $3ab^{49}$

8. a. $2u^6v^{10}\sqrt[5]{2u^2v^4}$ b. $3z^6\sqrt[5]{2w^4z^2}$

9. Does your verification prove that the answer is right?

10. For instance, $\sqrt{9+16} = \sqrt{25} = 5$, whereas $\sqrt{9}+\sqrt{16} = 3+4 = 7$. Note that these numbers constitute a *counterexample*.

11. a. $\left(7^{1/4}\right)^{1/2} = 7^{1/8} = \sqrt[8]{7}$ b. $\sqrt[6]{10}$

c. $\left(x^{1/7}\right)^{1/5} = x^{1/35} = \sqrt[35]{x}$ d. $\sqrt[16]{n}$

e. $\left(\left(a^{1/3}\right)^{1/2}\right)^{1/5} = a^{\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{5}} = a^{1/30} = \sqrt[30]{a}$ f. $\sqrt[24]{x}$

12. a. $\sqrt[6]{x^2} = x^{2/6} = x^{1/3} = \sqrt[3]{x}$ b. $\sqrt[8]{n^6} = n^{6/8} = n^{3/4} = \sqrt[4]{n^3}$

c. $\sqrt{x}$ d. $\sqrt[3]{z}$ e. $\sqrt[4]{a^3}$ f. $\sqrt[3]{n^2}$ g. $\sqrt[15]{m^{13}}$ h. $\sqrt[8]{x^7}$

13. a. $\sqrt[24]{3}$; the problem and the answer are each about 1.046839.

14. a. $12\sqrt{x}$ b. As is c. $13\sqrt[3]{n}$ d. As is e. $68\sqrt{2}$

f. $5\sqrt[3]{3}$ g. $-3\sqrt{5}$ h. $32\sqrt[3]{7}$ i. $3\sqrt[3]{z^2}$

15. a. $\sqrt{84} = 2\sqrt{21}$ b. $\sqrt[3]{27} = 3$ c. $\sqrt{x^8} = x^4$

d. $\sqrt[3]{x^3y^{13}} = xy^4\sqrt[3]{y}$ e. $\sqrt{28} = 2\sqrt{7}$ f. $\sqrt[3]{125} = 5$

g. $\sqrt[4]{16} = 2$ h. $\sqrt{xyz}$ i. $\sqrt[3]{8} = 2$

16. a. $\sqrt{n}+n$ b. $x^2 - x\sqrt{y} + x\sqrt{y} - y = x^2 - y$

c. $4+4\sqrt{3}+3 = 7+4\sqrt{3}$ d. $\left(\sqrt{p}-q\right)\left(\sqrt{p}-q\right) = p - 2q\sqrt{p} + q^2$

e. $7 - 4 = 3$ f. $x^2 - 2x\sqrt{n} + n$ g. $a - c$

h. $2\left(\sqrt{3}+2\right) = 2\sqrt{3}+4$ i. $\left(2\sqrt{x}\right)^2 = 4x$

17. a. $\dfrac{2}{\sqrt{8}}\left[\dfrac{\sqrt{8}}{\sqrt{8}}\right] = \dfrac{2\sqrt{8}}{8} = \dfrac{4\sqrt{2}}{8} = \dfrac{\sqrt{2}}{2}$ b. $x\sqrt{x}$

c. $\dfrac{3}{5+\sqrt{2}}\left[\dfrac{5-\sqrt{2}}{5-\sqrt{2}}\right] = \dfrac{15-3\sqrt{2}}{25-2} = \dfrac{15-3\sqrt{2}}{23}$ d. $\dfrac{x+2\sqrt{xy}+y}{x-y}$

e. $\dfrac{\sqrt{3}}{3}$ f. 2 g. $4\sqrt{2}$ h. $\dfrac{\sqrt{xy}}{y}$

i. $\dfrac{5}{1+\sqrt{5}}\left[\dfrac{1-\sqrt{5}}{1-\sqrt{5}}\right] = \dfrac{5-5\sqrt{5}}{1-5} = \dfrac{5-5\sqrt{5}}{-4} = \dfrac{5-5\sqrt{5}}{-4}\left[\dfrac{-1}{-1}\right] = \dfrac{5\sqrt{5}-5}{4}$

The last step is optional -- it was done just to remove the negative sign from the bottom of the fraction.

j. $-3-2\sqrt{2}$ k. $\dfrac{7}{3\sqrt{2}}\left[\dfrac{\sqrt{2}}{\sqrt{2}}\right] = \dfrac{7\sqrt{2}}{3\cdot 2} = \dfrac{7\sqrt{2}}{6}$

l. $\frac{\sqrt{x}}{n}$ m. $\frac{7\sqrt{7}-7}{6}$ n. $\frac{a^2+2a\sqrt{b}+b}{a^2-b}$ o. $\frac{a\sqrt{c}}{bc}$ p. 1

18. c. and d. are valid -- a. and b. are bogus.

19. a. ± 4 b. -10 c. None

20. a. $\sqrt{35}$ b. $\sqrt{7}+\sqrt{5}$ c. $\frac{\sqrt{35}}{5}$ d. $\sqrt{7}-\sqrt{5}$
e. $\sqrt{2}+\sqrt[3]{2}$ f. $7\sqrt{7}$ g. $h+14\sqrt{h}+49$ h. $9-6\sqrt{t}+t$

21. $3x^2y^9z^{16}\sqrt[3]{4xz^2}$ **22**. a. $\sqrt[8]{x^4}$ b. $\sqrt[12]{x}$

23. a. $\frac{2\sqrt{35}+10}{5}$ b. $\frac{a-2\sqrt{ab}+b}{a-b}$ c. 2

24. a. 12 b. 0 c. $\frac{\sqrt{7}+7}{7}$ d. $3\sqrt{2}$

25. $2xy^6z^{12}\sqrt[4]{4x^3y^3z^2}$ **26**. $\sqrt[24]{x}$

27. x **28**. a. $2\sqrt{7}-5$ b. $\frac{a+2b\sqrt{a}+b^2}{a-b^2}$ c. -1

29. Do the arithmetic and it should work out.

30. a. T b. F c. F d. T e. T f. F g. T h. F i. F
j. F k. F l. T m. F n. T o. T p. F q. T r. F

The important thing
is to not stop questioning.

Albert Einstein